Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Friday 19 May 2023

Afternoon

Paper reference **8FM0/22**

Further Mathematics

Advanced Subsidiary

Further Mathematics options

22: Further Pure Mathematics 2

(Part of option A only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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P 7 2 8 0 8 A 0 1 1 6

Pearson

2.1: The Axioms For A Group | 2.2: Cayley Tables & Finite Groups | 2.3: Order & Subgroups

1. The operation * is defined on the set $G = \{0, 1, 2, 3\}$ by

$$x * y \equiv x + y - 2xy \pmod 4$$

(a) Complete the Cayley table below.

(2)

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

(b) Show that G is a group under the operation *

(You may assume the associative law is satisfied.)

(3)

(c) State the order of each element of G.

(2)

(d) State whether G is a cyclic group, giving a reason for your answer.

(1)

b. To prove if G is a group, must check the following:

→ Closure: all elements in Cayley table are in set G, so closed ✓

→ Identity: 0 is the identity ✓

→ Inverse:

x	0	1	2	3
x^{-1}	0	1	2	3

all elements have an inverse ✓

→ associativity: can assume satisfied (as Q states) ✓

All axioms are satisfied, ∴ G is a group under the operation * //

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Question 1 continued

c. $0^1 = 0$ | $1^1 = 1$, $1^2 = 0$, $1^3 = 1$ | $2^1 = 2$, $2^2 = 0$, $2^3 = 2$ | $3^1 = 3$, $3^2 = 0$, $3^3 = 3$

x^3 returns back to x^1 so order 2 for these elements

element	0	1	2	3
order	1	2	2	2

d. There is no element with order 4 $\therefore G$ is not a cyclic group.

Every element is its own inverse $\therefore$ no group generator $\therefore G$ is not a cyclic group.

(Total for Question 1 is 8 marks)

P 7 2 8 0 8 A 0 3 1 6

2. A linear transformation $T:\mathbb{R}^2 \rightarrow \mathbb{R}^2$ is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} 5 & 1 \\ k & -3 \end{pmatrix}$$

where k is a constant.

Given that matrix **M** has a repeated eigenvalue,

(a) determine

(i) the value of k

(ii) the eigenvalue.

(6)

(b) Hence determine a Cartesian equation of the invariant line under T.

only looking for 1 eqⁿ

(2)

ai. characteristic eqⁿ: $\det(M-\lambda I)=0$

$$M-\lambda I = \begin{pmatrix} 5 & 1 \\ k & -3 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 1 \\ k & -3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$= \begin{pmatrix} 5-\lambda & 1 \\ k & -3-\lambda \end{pmatrix}$$

$$\det\left[\begin{pmatrix} 5-\lambda & 1 \\ k & -3-\lambda \end{pmatrix}\right] = 0$$

$\det(M)$ where $M=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\det(M) = (a)(d)-(b)(c)$

$(5-\lambda)(-3-\lambda)-(1)(k)=0$

$(\lambda^2-2\lambda-15)-(k)=0$

$\lambda^2-2\lambda-15-k=0$

If there is a repeated eigenvalue, there is a repeated root for characteristic eqⁿ.

$\therefore\ b^2-4ac=0$

$(-2)^2-(4)(1)(-15-k)=0$

$4-(-60-4k)=0$

$4+60+4k=0$

$4k=-64$

$k=-16$ //

Question 2 continued

ii. $\lambda^2 - 2\lambda - 15 + 16 = 0$

$\lambda^2 - 2\lambda + 1 = 0$

$(\lambda - 1)^2 = 0$

$\lambda = 1$

eigenvalue $= 1$ //

b. $M\begin{pmatrix} x \\ y \end{pmatrix} = \lambda\begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} 5 & 1 \\ -16 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 1\begin{pmatrix} x \\ y \end{pmatrix}$

$5x + y = x$ (1)

$-16x - 3y = y$ (2)

(1): $5x + y = x$

$y = -4x$

(2) $-16x - 3y = y$

$-16x = 4y$

$-4x = y$

both ① and ② give same eq^n

$\therefore\ y = -4x$ //

3. A complex number z is represented by the point P on an Argand diagram.

Given that

$$\arg\left(\frac{z-4-\mathrm{i}}{z-2-7\mathrm{i}}\right)=\frac{\pi}{2}$$ ← LOCUS EXPLAINED @ END OF Q

(a) sketch the locus of P as z varies,

(2)

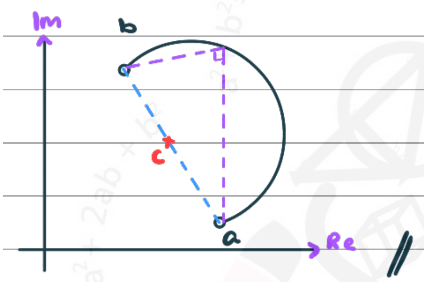
(b) determine the exact maximum possible value of $|z|$

(5)

a. $\arg\left(\frac{z-(4+i)}{z-(2+7i)}\right)=\frac{\pi}{2}$

$a = 4+i$

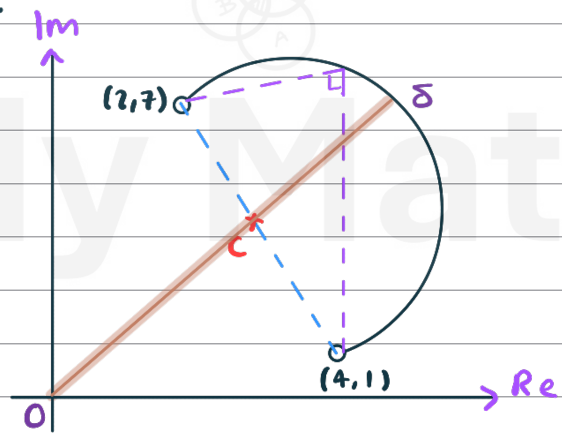
$b = 2+7i$



→ as $\theta = \frac{\pi}{2}$, the locus is a semicircle

→ arc drawn anticlockwise from $4+i$ to $2+7i$

b.



→ The brown line represents $|z|_{max}$

→ Let the furthest point on circle be δ.

→ Let C be centre of circle.

$|z|_{max} = |OC| + |C\delta|$

$|OC|$: need centre to calculate

$|C\delta|$: radius of circle

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Question 3 continued

Because hypotenuse of right-angled triangle is diameter of triangle's circumcircle

we also know: (1) radius of circumcircle is half length of hypotenuse

(2) centre of circumcircle is midpoint of hypotenuse

We will use (2) to find centre first:

Midpoint AB (centre) $= \left(\frac{4+2}{2}, \frac{1+7}{2}\right) = (3,4)$

$|OC| = \sqrt{(3-0)^2+(4-0)^2} = 5$

Use (1) to find radius

└ length $|AB| \div 2$

$|AB| = \sqrt{(4-2)^2+(1-7)^2} = 2\sqrt{10}$

$|AB| \div 2 = 2\sqrt{10} \div 2 = \sqrt{10}$

radius

$|CS| = \sqrt{10}$

$|z|_{max} = |OC| + |CS|$

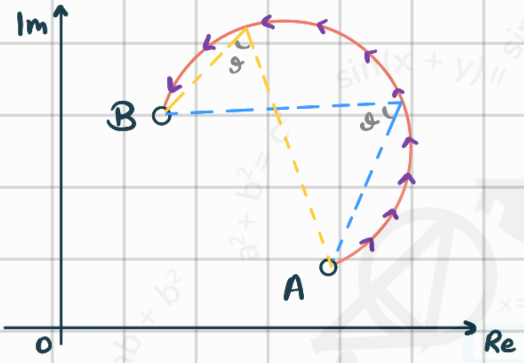
$|z|_{max} = 5 + \sqrt{10}$ //

(Total for Question 3 is 7 marks)

The locus of points z that satisfy $\arg\left(\frac{z-a}{z-b}\right) = \theta$

where $\theta \in \mathbb{R}$, $\theta > 0$ and $a, b \in \mathbb{C}$, is an arc of a circle with endpoints A and B representing the complex no.s a and b, respectively.

The locus is the arc of a circle drawn anticlockwise from A to B.



$\arg\left(\frac{z-a}{z-b}\right) = \theta$

if $\theta < \frac{\pi}{2}$ then the locus is a major arc of a circle
if $\theta = \frac{\pi}{2}$ then the locus is a semi-circle.
if $\theta > \frac{\pi}{2}$ then the locus is a minor arc of a circle

4. A student takes out a loan for £1000 from a bank.

The bank charges 0.5% monthly interest on the amount of the loan yet to be repaid.

At the end of each month

- the interest is added to the loan
- the student then repays £50

Let U_n be the amount of money owed n months after the loan was taken out.

The amount of money owed by the student is modelled by the recurrence relation

$$U_n = 1.005U_{n-1} - A \qquad U_0 = 1000 \qquad n \in \mathbb{Z}^+$$

where A is a constant.

(a) (i) State the value of the constant A.

(ii) Explain, in the context of the problem, the value 1.005

(2)

Using the value of A found in part (a)(i),

(b) solve the recurrence relation

$$U_n = 1.005U_{n-1} - A \qquad U_0 = 1000 \qquad n \in \mathbb{Z}^+$$

(5)

(c) Hence determine, according to the model, the number of months it will take to completely repay the loan.

(2)

ai. Student repays £50, $\therefore A = 50$

ii. Interest rate is 0.5% so multiplier 1.005

b. $u_n = 1.005\,u_{n-1} - 50$

Homogenous part (C.F.)

$u_n = 1.005\,u_{n-1}$

$u_n = c(1.005)^n$

Non-homogenous part (P.I.)

$u_n = \lambda$

$u_{n-1} = \lambda$

Sub into u_n eqn and solve for λ

$\lambda = 1.005\lambda - 50$

$50 = 0.005\lambda$

$\lambda = 10000$

Question 4 continued

gen sol^n: C.F. + P.I

gen sol^n: $u_n = c(1.005)^n + 10000$

$u_0 = c(1.005)^0 + 10000 = 1000$

$c + 10000 = 1000$

$c = -9000$

Particular sol^n: $u_n = 10000 - 9000(1.005)^n$ //

c. set $u_n = 0$ and solve for n

$0 = 10000 - 9000(1.005)^n$ } $+9000(1.005)^n$ on both sides

$9000(1.005)^n = 10000$ } $\div 9000$ on both sides

$(1.005)^n = 10/9$

$n = \log_{(1.005)}(10/9) = 21.1247396$ months } take logs on both sides and evaluate to find n

∴ will take 22 months to repay //

1.3: Modular Arithmetic | 1.4: Divisibility Tests

5. (i) Making your reasoning clear and using modulo arithmetic, show that

$$214^6 \text{ is divisible by } 8$$

(3)

(ii) The following 7-digit number has four unknown digits

$\boxed{a}\,5\,\boxed{b}\,8\,\boxed{a}\,\boxed{b}\,0$

Given that the number is divisible by 11

(a) determine the value of the digit a.

only 1 value

(2)

Given that the number is also divisible by 3

(b) determine the possible values of the digit b.

more than 1 value for b

(3)

i. firstly simplify 214 mod 8.

$214 \equiv 6 \bmod 8$ or $\underbrace{-2 \bmod 8}$

we will use this as it is easier to calculate moving forward

$214 \equiv -2 \bmod 8$ raise both sides to the power 6

$(214)^6 \equiv (-2)^6 \bmod 8$

what we are trying to work out

$(-2)^6 \bmod 8 \equiv 64 \bmod 8$

$\equiv 0 \pmod 8$ no remainder left so is divisible

$\therefore 214^6$ is divisible by 8.

iia. If a no. is divisible by 11, if the sum of alternating digits is divisible by 11.

$a - 5 + b - 8 + a - b + 0$

$= 2a - 13$

$\underbrace{1 \leq a \leq 9}$

sub in all these values of a and

$\therefore a = 1$

$a = 1$ $2(1) - 13 = -11$ 1 will be divisible by 11.

$a = 2$ $2(2) - 13 = -9$

$a = 3$ $2(3) - 13 = -7$

$a = 4$ $2(4) - 13 = -5$

$a = 5$ $2(5) - 13 = -3$

$a = 6$ $2(6) - 13 = -1$

$a = 7$ $2(7) - 13 = 1$

$a = 8$ $2(8) - 13 = 3$

$a = 9$ $2(9) - 13 = 5$

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Question 5 continued

b. An integer is divisible by 3 iff the sum of it's digits is divisible by 3.

$1 + 5 + b + 8 + 1 + b + 0 = 3n$

$= 2b + 15$

$\underbrace{0 \leq b \leq 9}$

Sub in all these values – multiple values should be divisible by 3.

← means that for this b value, $3 \mid 2b+15$

$b=0 \quad 2(0)+15 = 15$

$b=1 \quad 2(1)+15 = 17$

$b=2 \quad 2(2)+15 = 19$

$b=3 \quad 2(3)+15 = 21$

$b=4 \quad 2(4)+15 = 23$

$b=5 \quad 2(5)+15 = 25$

$b=6 \quad 2(6)+15 = 27$

$b=7 \quad 2(7)+15 = 29$

$b=8 \quad 2(8)+15 = 31$

$b=9 \quad 2(9)+15 = 33$

$\therefore \; b = 0, 3, 6, 9$ //